

# Computer Lab - Geodynamics Course Computation of density, gravity and pressure in planetary interiors

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**Please read this document in its entirety before starting to code and debug. Look at all the code templates provided to you too. These assignments are voluntarily designed so as to leave much more freedom to the student than those of ProgMod.**

## 1 Introduction

In this computerlab you will investigate the radial distribution of density, gravity and pressure in a spherically symmetric planet model. For the special case of a parameterized density model this problem can be solved by evaluating the integral expressions for the gravity and pressure in terms of the given density distribution, derived in the lecture notes. In case of a self-compressing planet model, the density is a function of the local pressure and the density profile has to be solved from a differential equation and a physical model or parameterization for the compressibility is required. In the assignment of section 2 you will investigate cases with parameterized density distributions and compare different parameterizations which have been constrained by the total mass and moment of inertia of the Earth, with the PREM model. Section 3 deals with an investigation of density distributions for compressible earth models. In section 4 the relation between planet radius and planetary mass is determined for a compressible model of a massive exoplanet.

## 2 Parameterized density models

**The goal of this first assignment is to write/update the Fortran file program.f90 which implements various radial density tables for simple parameterized density distributions, specified hereafter.**

Once a table of a density profile  $\rho(r)$  is available, the corresponding gravity  $g(r)$  and pressure  $P(r)$  can be computed from it by numerical integration. The provided `compute_XXX` subroutines carry out such a numerical integration, and will therefore be used to compute tables of the radial distribution of gravity and pressure in the interior of a spherically symmetric planet.

The program also includes the `analytical_solution` subroutine in which you are to implement the corresponding analytical expressions for the gravity  $g$  and pressure  $P$  profiles <sup>1</sup>.

All computed profile tables must be written to file for later use in computations and plotting.

The Makefile and all the fortran files can be found online on my website.

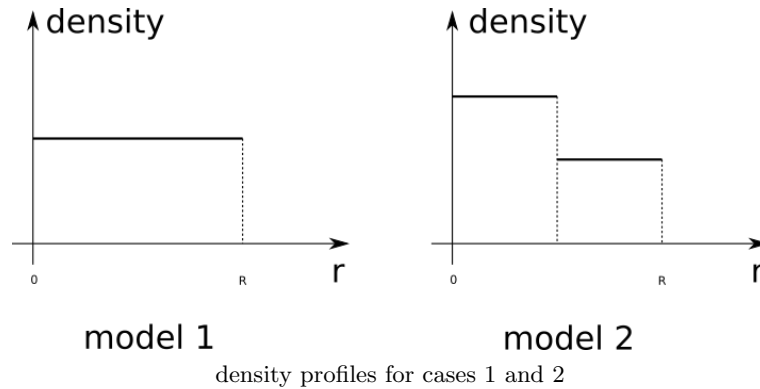
### 2.1 Modelling experiments

The special cases of density parameterizations to be investigated include the following models:

- A uniform density model (`model=1`), resulting in a gravity acceleration that is a linear function of the radial coordinate and a quadratic pressure distribution. Define the uniform density in this model from the given total mass  $M$  and outer radius  $R$  which are set to the corresponding Earth values.
- A two layer density model (`model=2`), consisting of a uniform core and mantle with given radius  $R_c$  and  $R$  respectively, equal to those of the PREM. The remaining parameters of this model are the uniform core and mantle densities  $\rho_c$  and  $\rho_m$ , which are constrained by the total mass,  $M$  and moment of inertia,  $I$ . Express  $\rho_c$  and  $\rho_m$  in terms of  $M$  and  $I$ ,  $R_c$  and  $R$ . Use  $M$  and  $I$  values that are representative for the Earth.

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<sup>1</sup>In order to validate the implementation of the computational schemes for gravity and pressure the outcome will be compared to the corresponding radial profiles of the parameterized density models that can be computed analytically. This has to be done only for the two simpler models of a uniform planet (see model m0 below) and a 2-layer planet (see model m1 below) for which expressions for the gravity and pressure profiles are given in the lecture notes.



- A three layer density model (`model=3`), consisting of a uniform core with given (PREM) radius  $R_c$  and a mantle, divided at a depth of 660 km in two uniform shells, corresponding to the upper and lower mantle. Assuming the value of the upper mantle density  $\rho_{um}$  to be known, the remaining density parameters,  $\rho_{lm}$  and  $\rho_c$  of the lower mantle and core can be expressed in terms of the global density constraints  $M$  and  $I$ . To this end apply the formulation for a two-layer Earth model and consider the related problem of a model with the upper mantle stripped off, using modified values  $M'$  and  $I'$  for the mass and moment of inertia with  $M = M_{um} + M'$  and  $I = I_{um} + I'$ , where  $M_{um}$  and  $I_{um}$  are the mass and moment of inertia of the upper mantle. Try different values of  $\rho_{um}$  and include results in your report of a case where the pressure at the upper/lower mantle boundary P660 equals the PREM value of 23.9 GPa.
- `model=4`: read in the density values of the PREM model and use the same methodology as before to arrive at the mass, gravity and pressure profiles. Compare the last two with the real PREM data and discuss.

## 2.2 Program structure and further instructions

- Use the `make` command to compile and link all fortran files so as to produce the `grav` executable.
- Use double precision variables for real numbers in your program to avoid numerical overflow problems in computations based on the dimensional expressions given in the lecture notes. This can be done by using the type declaration `real(8)` instead of `real`.
- The resulting tables of the radial profiles are to be written to file for comparison with corresponding profiles of the Preliminary Reference Earth Model (PREM).
- PREM data are made available to you in the `PREMdata` folder. These data are not to be read in by a fortran program except for model 4. They are to be plotted against your results with `gnuplot`.
- All plots are to be done with the `gnuplot` program and exported as pdf files. Axis labels and captions are *not* optional.
- The `select case` command was not part of ProgMod. Please have a look there <http://www.cs.mtu.edu/~shene/COURSES/cs201/NOTES/chap03/select.html>
- A single report per student pair is expected by March 11th, 11.59pm. Reports should be sent per email to `c.thieulot@uu.nl` in *pdf* format. I have also a strong preference for documents produced with L<sup>A</sup>T<sub>E</sub>X. Please try <https://www.overleaf.com/> or <https://www.sharelatex.com/>