GEO3-1313: Geodynamica

C. Thieulot (c.thieulot@uu.nl)

February 2022

organisation (1)

- Monday 7/02 - morning
- Wednesday 9/02 - morning
- Monday 14/02 - morning
- Wednesday 16/02 - morning
- Monday 21/02 - morning
- Wednesday 23/02 - morning
- Monday 28/03 - morning
- Wednesday 2/03 - morning

Exam : Monday 7/03

organisation (2)

- Computer practicals (python !)
- 6-7 lectures + 4 computer sessions

organisation (2)

- Computer practicals (python !)
- 6-7 lectures + 4 computer sessions
- 50% credits on exam, 50% lab report

organisation (2)

- Computer practicals (python !)
- 6-7 lectures + 4 computer sessions
- 50% credits on exam, 50% lab report
- guest lecture(s) at the end

Fieldstone

What is it ?

Fieldstone

What is it ? I am not sure anymore :)

Fieldstone

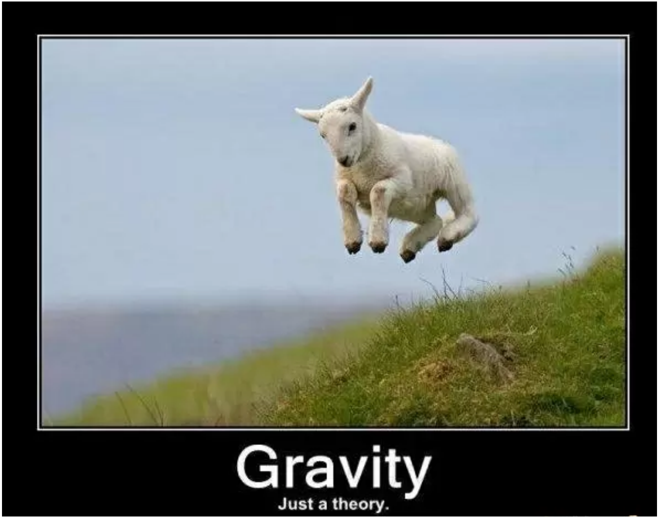
What is it ? I am not sure anymore :)

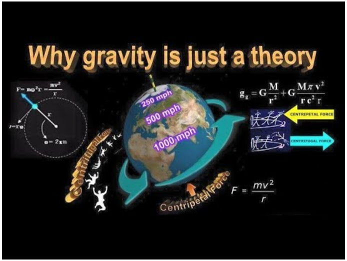
- A single source of information for Geophysics and Computational Geodynamics
- Consistent notations throughout
- Used for GEO3-1313 (Geodynamics), GEO4-1416 (Mantle Dynamics) and GEO4-1427 (Computational Geophysics)
- Enormous bibliography ($>$ 4200 refs) organised by topics
- Python codes illustrating many features found in state-of-the-art codes
- Dynamic document, continuously updated
- Open source `https://cedrict.github.io/`
- Chapt 9 $=$ syllabus for GEO3-1313

Please give me feedback ! typos ? structure ? grammar ? figures ? etc ...

gravity... ?

gravity... ?





https://www.youtube.com/watch?v=VNqNnUJVcVs

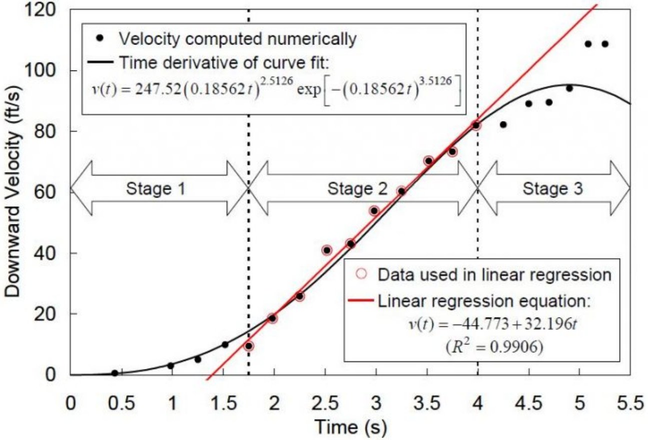


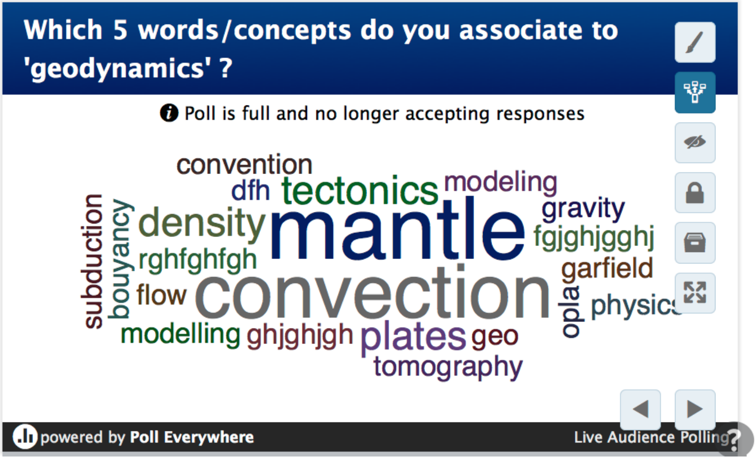
Figure 12–77 Downward velocity of north face roofline as WTC 7 began to collapse.

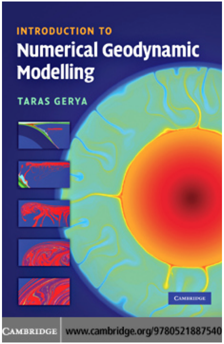
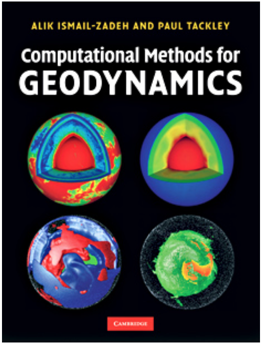
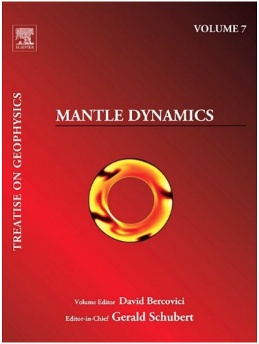
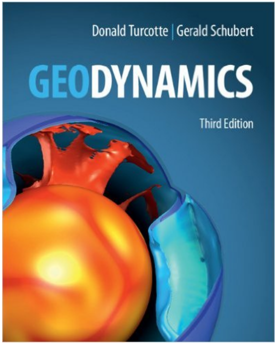
Lecture 1

Fielstone Chapter 9.

- Early models of the Earth density
- Moment of inertia
- Density, gravity and pressure
- Gravity field

Geodynamics




INTRODUCTION TO
Numerical Geodynamic Modelling
TARAS GERYA
CAMBRIDGE
www.cambridge.org/9780521887540

ALIK ISMAIL-ZADEH AND PAUL TACKLEY
Computational Methods for
GEODYNAMICS
CAMBRIDGE

VOLUME 7
TREATISE ON GEOPHYSICS
MANTLE DYNAMICS
Volume Editor David Bercovici
Editor-in-Chief Gerald Schubert

Donald Turcotte | Gerald Schubert
GEODYNAMICS
Third Edition

Home › Books & Journals › Journal of Geodynamics

Journal of Geodynamics

Editor-in-Chief: I.M. Artemieva
View full editorial board

Supports Open Access

ISSN: 0264-3707

Original research papers, including 'letters', as well as topical reviews are invited on :

Earth rotation Rheology and mineral properties of the deep earth, physical properties of rocks and their dependence on pressure, temperature and chemical composition Upper mantle - lower mantle ; lithosphere - asthenosphere Mantle convection, hot spots and plumes, heat flow and the thermo-mechanical evolution of the earth Plate kinematics, plate tectonics and plate dynamics, driving mechanisms Stress field ; horizontal and vertical crustal movements Evolution of continents and oceans, including the formation and destruction of oceanic lithosphere, orogenic processes and basin evolution Crust-mantle interaction, chemical recycling Sea surface and ocean bottom topography, including variations of sea level Dynamic interpretation and modelling of potential fields, including isostasy, glacial isostasy Magma formation, differentiation, transport and emplacement, including modelling of volcanic eruptions Dynamic consequences of natural events, including source dynamics, seismic modelling, seismo-tectonics, modelling of earthquakes, impacts Integrated models and non-linear processes.

Geodynamics numerical modelling 101

a bit of history (1)

- The concept of a spherical Earth dates back to around the 6th century BC, when it was mentioned in ancient Greek philosophy, but remained a matter of philosophical speculation until the 3rd century BC, when Hellenistic astronomy established the spherical shape of the earth as a physical given.
- The paradigm was gradually adopted throughout the Old World during Late Antiquity and the Middle Ages.
- A practical demonstration of Earth's sphericity was achieved by Magellan and Elcano's expedition's circumnavigation (1519-1522).

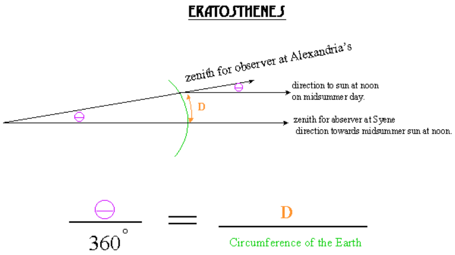
a bit of history (1)

- The concept of a spherical Earth dates back to around the 6th century BC, when it was mentioned in ancient Greek philosophy, but remained a matter of philosophical speculation until the 3rd century BC, when Hellenistic astronomy established the spherical shape of the earth as a physical given.
- The paradigm was gradually adopted throughout the Old World during Late Antiquity and the Middle Ages.
- A practical demonstration of Earth's sphericity was achieved by Magellan and Elcano's expedition's circumnavigation (1519-1522).

$\rightarrow$ What about the **interior** ?

a bit of history (2)

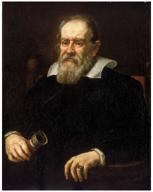
In approximately 230 BC, the Greek mathematian, **Eratosthenes** calulated the radius of the Earth. He compared the shadows in the wells during the summer solstice and obtained the value $R \sim 6.38 \times 10^6 m$.



https://youtu.be/AWtfJ2D1ONM

a bit of history (3)

In the 16th century, **Galileo** determined the acceleration due to the force of gravity near the surface of the Earth and obtained 9.8 m/s^2.




In his famous experiment dropping balls from the Tower of Pisa, and later with careful measurements of balls rolling down inclines, Galileo showed that gravity accelerates all objects at the same rate. This was a major departure from Aristotle's belief that heavier objects accelerate faster.

 `https://youtu.be/03SPBXALJZI`

Galileo postulated air resistance as the reason that lighter objects may fall more slowly in an atmosphere. Galileo's work set the stage for the formulation of Newton's theory of gravity.

a bit of history (4)

Sir Isaac **Newton** (1642-1726) greatly contributed to the study of physics and therefore, his efforts determined the mass of the Earth. His law of gravity and second law of motion are used together to obtain a value for the mass of our planet.

First law:	When viewed in an inertial reference frame, an object either remains at rest or continues to move at a constant velocity, unless acted upon by a force.[2][3]
Second law:	The vector sum of the forces $\mathbf{F}$ on an object is equal to the mass m of that object multiplied by the acceleration vector $\mathbf{a}$ of the object: $\mathbf{F} = m\mathbf{a}$.
Third law:	When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.

Newton's law of gravity formulates the gravitational force that two masses exert on each other and is given by

$$F = \frac{\mathcal{G} m M}{r^2}$$

M an m are the two masses, r is the separation between them, and $\mathcal{G}$ is the universal gravitational constant.

a bit of history (4)

The value of $\mathcal{G}$ which was calculated by Henry **Cavendish** in 1798 :
$\mathcal{G} = 6.67 \times 10^{-11}\ \mathrm{m}^3/(\mathrm{kg.s}^2)$.

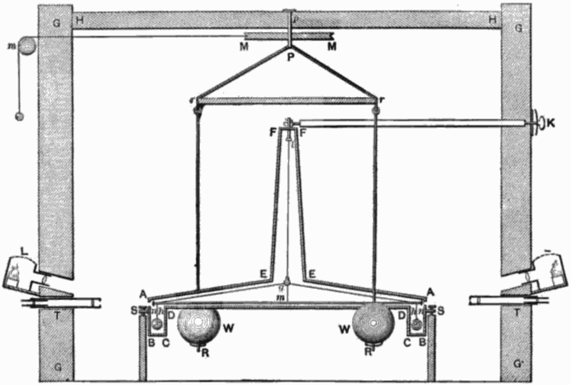


Fig. 1

https://youtu.be/2PdiUoKa9Nw

a bit of history (5)

If we assumed that M is the mass of the Earth, and m is the mass of an object on the surface of the Earth, we can solve for M by equating Newton's Law of Gravity with his second law of motion

$$F = m \cdot a$$

We have :

$$F = GmM/r^2 = m \cdot a \rightarrow GM/r^2 = a$$

Solving for M, the mass of the Earth, and using

$$\begin{aligned} a &= 9.8 m/s^2 \\ R &= 6.38 \times 10^6 m \\ G &= 6.67 \times 10^{-11} m^3/(kg \cdot s^2) \end{aligned} \tag{1}$$

we obtain :

$$M = aR^2/\mathcal{G} = 5.98 \times 10^{24} kg.$$

A little problem

What is 'density'?

A little problem

What is 'density'? The (volumetric mass) density of a substance is its mass per unit volume.

A little problem

What is 'density' ? The (volumetric mass) density of a substance is its mass per unit volume.
Units ?

A little problem

What is 'density'? The (volumetric mass) density of a substance is its mass per unit volume.
Units? kg/m^3

A little problem

What is 'density' ? The (volumetric mass) density of a substance is its mass per unit volume.
Units ? kg/m^3
Ballpark figure of crustal rocks density ?

A little problem

What is 'density' ? The (volumetric mass) density of a substance is its mass per unit volume.
Units ? kg/m^3
Ballpark figure of crustal rocks density ? around 2500-3000 kg/m^3.

A little problem

What is 'density' ? The (volumetric mass) density of a substance is its mass per unit volume.
Units ? kg/m^3
Ballpark figure of crustal rocks density ? around 2500-3000 kg/m^3.
Volume of the Earth ?

A little problem

What is 'density' ? The (volumetric mass) density of a substance is its mass per unit volume.
Units ? kg/m^3
Ballpark figure of crustal rocks density ? around 2500-3000 kg/m^3.
Volume of the Earth ? $V = \frac{4}{3}\pi R^3$

A little problem

What is 'density' ? The (volumetric mass) density of a substance is its mass per unit volume.
Units ? kg/m^3
Ballpark figure of crustal rocks density ? around 2500-3000 kg/m^3.
Volume of the Earth ? $V = \frac{4}{3}\pi R^3$

$$\langle \rho \rangle = \frac{M}{V} \qquad \Rightarrow \qquad M = \langle \rho \rangle \, \frac{4}{3}\pi R^3 \sim 3 \times 10^{24}\text{kg}$$

A little problem

What is 'density' ? The (volumetric mass) density of a substance is its mass per unit volume.
Units ? kg/m^3
Ballpark figure of crustal rocks density ? around 2500-3000 kg/m^3.
Volume of the Earth ? $V = \frac{4}{3}\pi R^3$

$$\langle\rho\rangle = \frac{M}{V} \qquad \Rightarrow \qquad M = \langle\rho\rangle \, \frac{4}{3}\pi R^3 \sim 3 \times 10^{24}\text{kg}$$

We have just established that

$$M = 5.98 \times 10^{24} kg.$$

A little problem

What is 'density' ? The (volumetric mass) density of a substance is its mass per unit volume.
Units ? kg/m^3
Ballpark figure of crustal rocks density ? around 2500-3000 kg/m^3.
Volume of the Earth ? $V = \frac{4}{3}\pi R^3$

$$\langle \rho \rangle = \frac{M}{V} \qquad \Rightarrow \qquad M = \langle \rho \rangle \, \frac{4}{3}\pi R^3 \sim 3 \times 10^{24}\mathrm{kg}$$

We have just established that

$$M = 5.98 \times 10^{24}\, kg.$$

Large discrepancy !

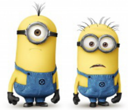
A little problem

What is 'density' ? The (volumetric mass) density of a substance is its mass per unit volume.
Units ? kg/m^3
Ballpark figure of crustal rocks density ? around 2500-3000 kg/m^3.
Volume of the Earth ? $V = \frac{4}{3}\pi R^3$

$$\langle \rho \rangle = \frac{M}{V} \qquad \Rightarrow \qquad M = \langle \rho \rangle \, \frac{4}{3}\pi R^3 \sim 3 \times 10^{24}\text{kg}$$

We have just established that

$$M = 5.98 \times 10^{24} kg.$$

 Large discrepancy !
→ Earth materials must have higher density at depth !

A little problem

What is 'density' ? The (volumetric mass) density of a substance is its mass per unit volume.
Units ? kg/m^3
Ballpark figure of crustal rocks density ? around 2500-3000 kg/m^3.
Volume of the Earth ? $V = \frac{4}{3}\pi R^3$

$$\langle \rho \rangle = \frac{M}{V} \qquad \Rightarrow \qquad M = \langle \rho \rangle \, \frac{4}{3}\pi R^3 \sim 3 \times 10^{24} \mathrm{kg}$$

We have just established that

$$M = 5.98 \times 10^{24} kg.$$

 Large discrepancy !

$\rightarrow$ Earth materials must have higher density at depth !
$\rightarrow$ Start simple : radial density distribution ? $\rho = \rho(r)$?

THE MASS AND MOMENT OF INERTIA OF THE EARTH

BARBARA ROMANOWICZ and KURT LAMBECK

Institut de Physique du Globe, Université Paris 6, 75230 Paris (France)

(Received April 5, 1977; revised and accepted May 13, 1977)

Romanowicz, B. and Lambeck, K., 1977. The mass and moment of inertia of the earth. Phys. Earth Planet. Inter., 15: P1–P4.

Recent revisions of geodetic and astronomical constants by the International Association of Geodesy and the International Astronomical Union lead to improved values for the earth's mass and moment of inertia. Corrections to be applied to these values before they are used as constraints in the inversion of seismic data are discussed.

Two constants frequently encountered in solid earth geophysics are the mass and the moment of inertia of the earth. Together these parameters provide constraints on the radial density distribution that any seismic model of the earth must satisfy. The mass M of the earth is not measured directly but the product GM is, where G is the gravitational constant. The most precise determination of this product now comes from the analysis of spacecraft accelerations and the precision with which M can be determined is limited by the precision with which G is known. The normalized moment of inertia is I/MR^2 where R is the radius of a sphere with the same volume as the earth and I is the moment of inertia of this sphere. The measured radius is not R but the equatorial radius R_e and the two are related by:

$$R = R_e (1-f/3)$$

where f is the flattening of the reference surface. I/MR^2 is estimated from the astronomical precession constant:

$$H = (C-A)/C$$

and from the second-degree zonal harmonics of the geopotential:

$$J_2 = (C-A)/\mathrm{MR}^2$$

A and C represent the equatorial and polar moments of the earth and:

$$I = (2A + C)/3$$

The flattening f of the geoid is related to J_2 by:

$$J_2 = \frac{2}{3}f - \frac{1}{3}m - \frac{1}{3}f^2 + \frac{2}{21}fm$$

with

$$m = \frac{\text{centrifugal force at equator}}{\text{gravity at equator}}$$

From the above definitions:

$$\frac{I}{MR^2} = (1 + \frac{2}{3}f - \frac{2}{3}H)\frac{J_2}{H}$$

Astronomical observations lead to the constant H through the equation (de Sitter, 1938):

Moment of inertia (1)

https://youtu.be/uyU25DdONjo

https://www.youtube.com/watch?v=fDJeVR0o__w (first 10 min)

Moment of inertia (2)

The polar moment of inertia is traditionally determined by combining measurements of spin quantities (spin precession rate or obliquity) and gravity quantities (coefficients in a spherical harmonics representation of the gravity field).

Moment of inertia (2)

The polar moment of inertia is traditionally determined by combining measurements of spin quantities (spin precession rate or obliquity) and gravity quantities (coefficients in a spherical harmonics representation of the gravity field).

Body	Value	Source
Earth	0.3307	[3]
Mars	0.3662 ± 0.0017	[4]
Mercury	0.346 ± 0.014	[5]
Moon	0.3929 ± 0.0009	[6]
Venus	unknown	

https://en.wikipedia.org/wiki/Moment_of_inertia_factor

Moment of inertia (2)

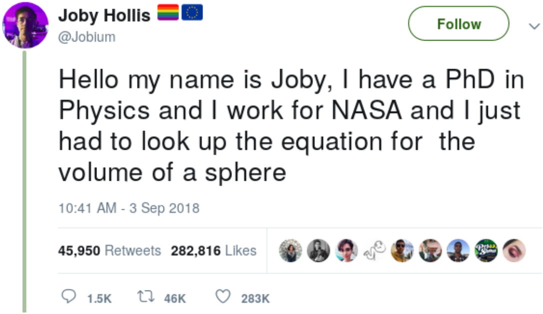
The polar moment of inertia is traditionally determined by combining measurements of spin quantities (spin precession rate or obliquity) and gravity quantities (coefficients in a spherical harmonics representation of the gravity field).

Body	Value	Source
Earth	0.3307	[3]
Mars	0.3662 ± 0.0017	[4]
Mercury	0.346 ± 0.014	[5]
Moon	0.3929 ± 0.0009	[6]
Venus	unknown	

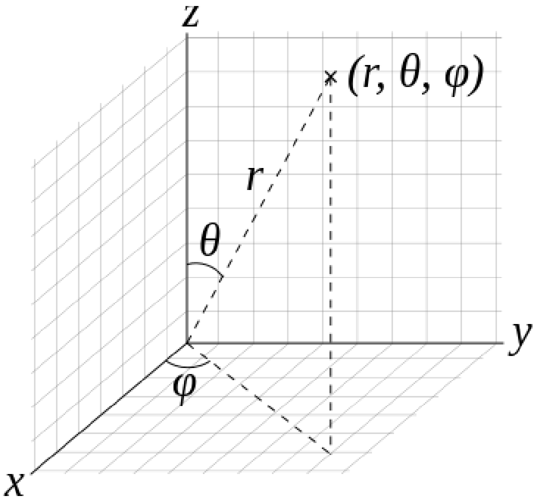
https://en.wikipedia.org/wiki/Moment_of_inertia_factor

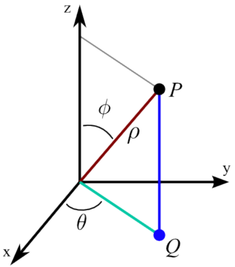
Double problem : internal structure $\rho(r)$ unknown, and I not well measured.

hem ...

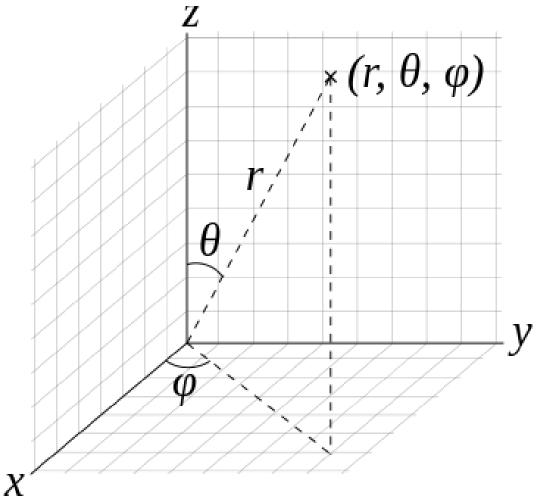

Joby Hollis
@Jobium
Follow
Hello my name is Joby, I have a PhD in Physics and I work for NASA and I just had to look up the equation for the volume of a sphere
10:41 AM - 3 Sep 2018
45,950 Retweets 282,816 Likes
1.5K
46K
283K

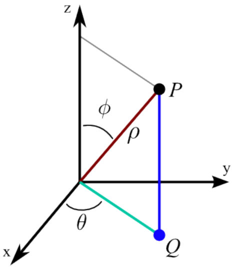
Spherical coordinates



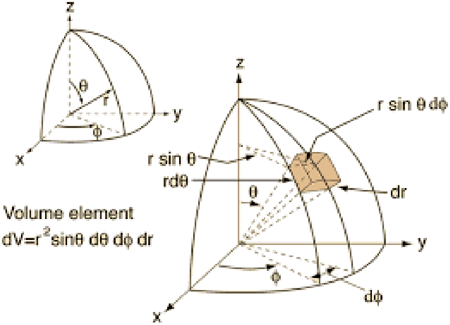


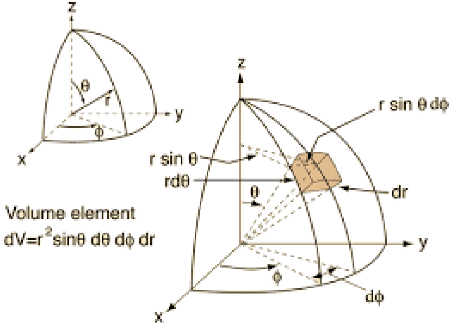
Spherical coordinates





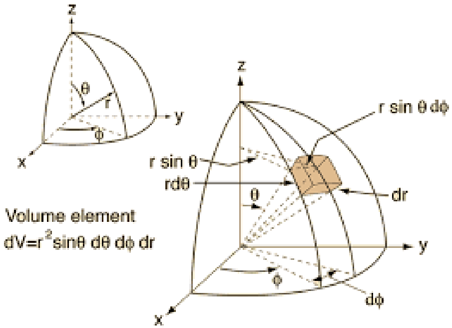
YES .... NO


z
θ
r
y
φ
x
z
r sin θ dφ
r sin θ
rdθ
dr
θ
Volume element
dV=r²sinθ dθ dφ dr
y
φ
dφ
x



Total volume :

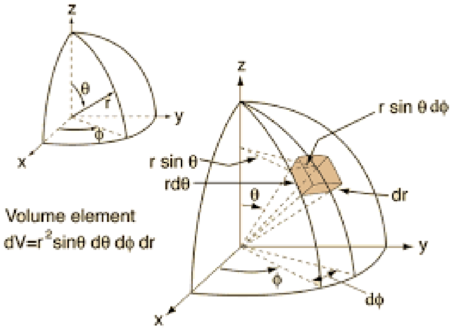
$$V = \int dV = \iiint r^2 \sin\theta \; dr d\theta d\phi$$



Total volume :

$$V = \int dV = \iiint r^2 \sin\theta \; dr d\theta d\phi$$

with $r \in [0, R]$, $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$



Total volume :

$$V = \int dV = \iiint r^2 \sin\theta \; dr d\theta d\phi$$

with $r \in [0, R]$, $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$
($\phi =$ longitude, θ=co-latitude)

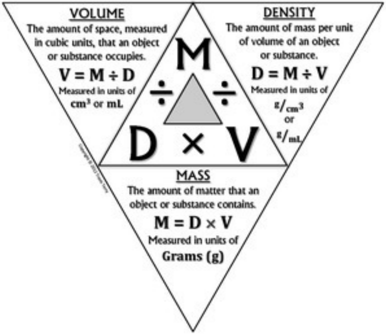
$$
\begin{aligned}
V &= \iiint r^2 \sin\theta \; dr d\theta d\phi \\
&= \left(\int r^2 dr\right)\left(\int \sin\theta d\theta\right)\left(\int d\phi\right)
\end{aligned}
$$

$$
\begin{aligned}
V &= \iiint r^2 \sin\theta \; dr d\theta d\phi \\
&= \left(\int r^2 dr\right)\left(\int \sin\theta d\theta\right)\left(\int d\phi\right) \\
&= \left(\int_0^R r^2 dr\right)\left(\int_0^\pi \sin\theta d\theta\right)\left(\int_0^{2\pi} d\phi\right) \\
&= \frac{1}{3}R^3 \cdot 2 \cdot 2\pi \\
&= \frac{4}{3}\pi R^3
\end{aligned}
$$

https://www.khanacademy.org/math/multivariable-calculus/integrating-multivariable-functions/x786f2022:polar-spherical-cylindrical-coordinates/a/triple-integrals-in-spherical-coordinates

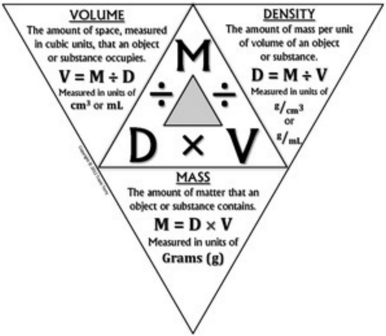
Density

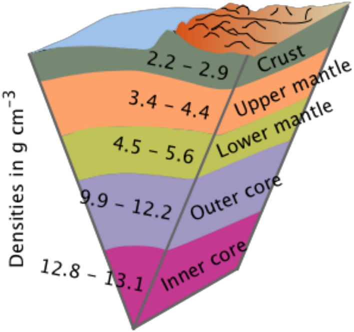
Mass density is expressed in kg/m^3.



Density

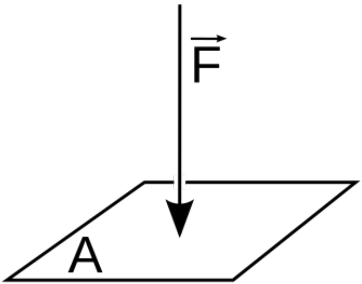
Mass density is expressed in kg/m^3.





Pressure

Pressure is expressed in Pa.
It represents a force (N) per unit area (m^2).



Pressure (2)

Lithostatic/hydrostatic pressure in a container :

- Height of container is H

Pressure (2)

Lithostatic/hydrostatic pressure in a container :

- Height of container is H
- mass density of fluid ρ_0

Pressure (2)

Lithostatic/hydrostatic pressure in a container :

- Height of container is H
- mass density of fluid ρ_0
- Steady state, no flow $\rightarrow \vec{v} = \vec{0}$.

Pressure (2)

Lithostatic/hydrostatic pressure in a container :

- Height of container is H
- mass density of fluid ρ_0
- Steady state, no flow $\rightarrow \vec{v} = \vec{0}$.
- The strainrate tensor is then nul $\rightarrow \dot{\epsilon} = \mathbf{0}$.

Pressure (2)

Lithostatic/hydrostatic pressure in a container :

- Height of container is H
- mass density of fluid ρ_0
- Steady state, no flow $\rightarrow \vec{v} = \vec{0}$.
- The strainrate tensor is then nul $\rightarrow \dot{\boldsymbol{\epsilon}} = \mathbf{0}$.
- The stress tensor then writes

$$\boldsymbol{\sigma} = -p\mathbf{1} + 2\mu\dot{\boldsymbol{\epsilon}} = -p\mathbf{1}$$

Pressure (2)

Lithostatic/hydrostatic pressure in a container :

- Height of container is H
- mass density of fluid ρ_0
- Steady state, no flow $\rightarrow \vec{v} = \vec{0}$.
- The strainrate tensor is then nul $\rightarrow \dot{\boldsymbol{\epsilon}} = \mathbf{0}$.
- The stress tensor then writes

$$\boldsymbol{\sigma} = -p\mathbf{1} + 2\mu\dot{\boldsymbol{\epsilon}} = -p\mathbf{1}$$

- the gravity vector is given by $\vec{g} = (0, -g)$

Pressure (3)

Momentum conservation equation :

$$\vec{\nabla} \cdot \boldsymbol{\sigma} + \rho \vec{g} = \vec{0}$$

The equation writes then

$$\begin{aligned} -\frac{\partial p}{\partial x} + \rho_0 g_x &= 0 \\ -\frac{\partial p}{\partial y} + \rho_0 g_y &= 0 \end{aligned}$$

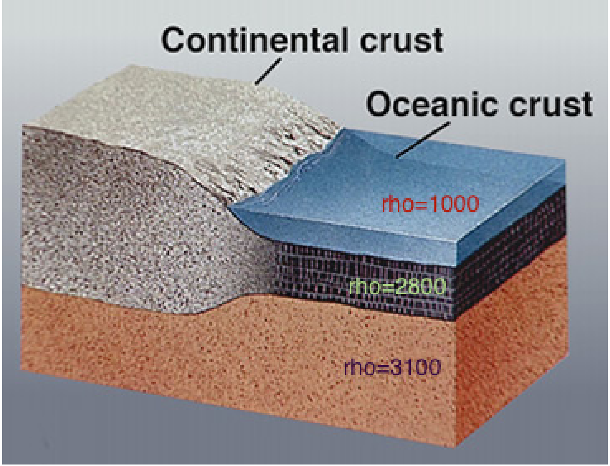
The first equation yields that the pressure is independent of x.
The second equation yields that the pressure is a linear function of the vertical coordinate y, i.e.

$$p(y) = -\rho g y + Constant$$

We request $p(y = H) = 0$ so that in the end

$$p(y) = \rho g (H - y)$$

Pressure (4)



$\rightarrow$ board

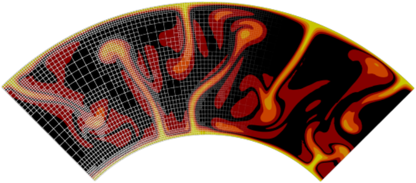
Gravity .... but why ?

Driving force in geodynamics

COMPUTATIONAL INFRASTRUCTURE FOR GEODYNAMICS (CIG)

ASPECT

Advanced Solver for Problems in Earth's ConvecTion



User Manual

Version 2.3.0-pre

(generated February 1, 2021)

Wolfgang Bangerth
Juliane Dannberg
Menno Fraters
Rene Gassmöller
Anne Glerum
Timo Heister
John Naliboff

The geodynamics equations

$$-\nabla\cdot\left[2\eta\left(\varepsilon(\mathbf{u})-\frac{1}{3}(\nabla\cdot\mathbf{u})\mathbf{1}\right)\right]+\nabla p=\rho\mathbf{g} \tag{2}$$

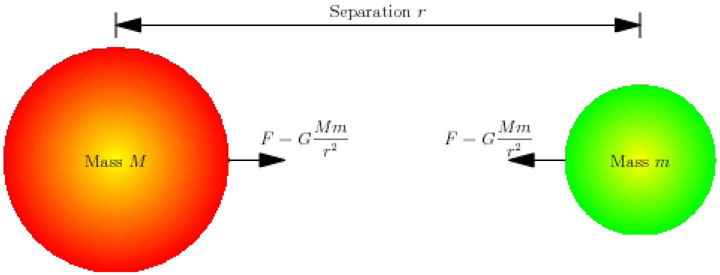
$$\nabla\cdot(\rho\mathbf{u})=0 \tag{3}$$

$$\begin{aligned}\rho C_p\left(\frac{\partial T}{\partial t}+\mathbf{u}\cdot\nabla T\right)-\nabla\cdot k\nabla T&=\rho H\\ &+2\eta\left(\varepsilon(\mathbf{u})-\frac{1}{3}(\nabla\cdot\mathbf{u})\mathbf{1}\right):\left(\varepsilon(\mathbf{u})-\frac{1}{3}(\nabla\cdot\mathbf{u}\right.\\ &+\alpha T\left(\mathbf{u}\cdot\nabla p\right)\\ &+\rho T\Delta S\left(\frac{\partial X}{\partial t}+\mathbf{u}\cdot\nabla X\right)\end{aligned} \tag{4}$$

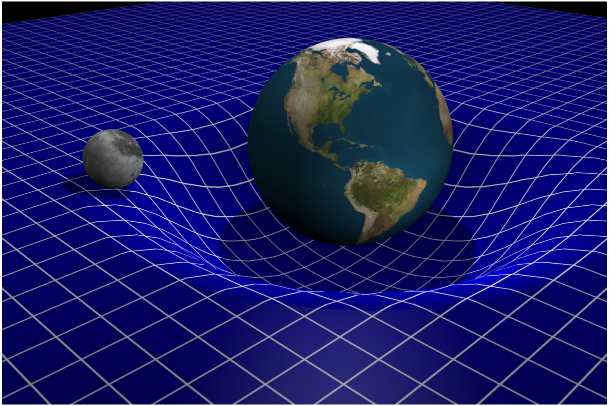
$$\frac{\partial c_i}{\partial t}+\mathbf{u}\cdot\nabla c_i=q_i \tag{5}$$

"buoyancy-driven flow"

Gravity (1)



Gravity (2)



 https://youtu.be/MTY1Kje0yLg

Gravity (2) - bis

"Why Gravity is NOT a Force" by Veritasium (17min)

https://www.youtube.com/watch?v=XRr1kaXKBsU

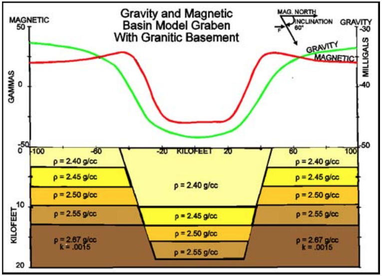
Gravity (2) - bis

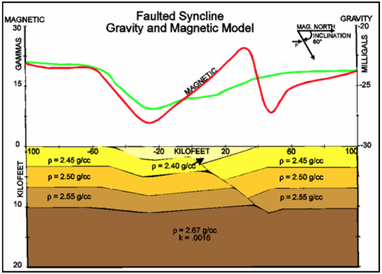
"Why Gravity is NOT a Force" by Veritasium (17min)

https://www.youtube.com/watch?v=XRr1kaXKBsU

Gravity (3)

Gravity measurements are an important part of geophysics :





Gravity (4)

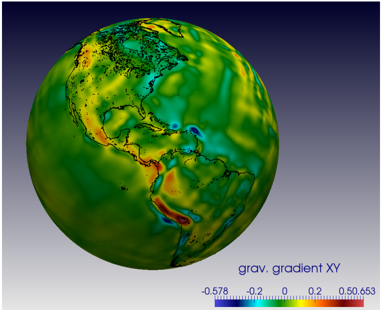
GRACE & GOCE satellites data : gravity and gravity gradient for the whole Earth with a $1^\circ \times 1^\circ$ resolution.

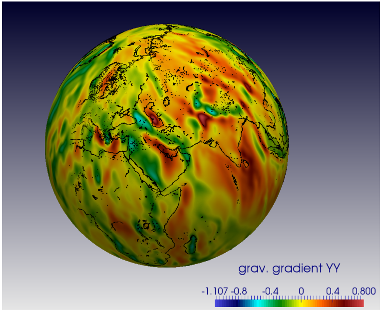
`https://youtu.be/qu-o75pe5GY`

Gravity (4)

GRACE & GOCE satellites data : gravity and gravity gradient for the whole Earth with a $1^\circ \times 1^\circ$ resolution.

https://youtu.be/qu-o75pe5GY





Let's talk units

- The SI units for (gravity) acceleration are $\mathrm{m\,s^{-2}}$. However in the context of gravity, we will rarely encounter these.
- The Gal is the commonly used unit in gravimetry :

$$0.01\mathrm{m\,s^{-2}} = 1\mathrm{Gal}$$

  and often measurements are given in mGal or μGal.
- As such, the acceleration due to Earth's gravity at its surface is 976 to 983 Gal, the variation being due mainly to differences in latitude and elevation.

Gravity model for Earth

The type of gravity model used for the Earth depends upon the degree of fidelity required for a given problem. For many problems such as aircraft simulation, it may be sufficient to consider gravity to be a constant, defined as:[3]

$g =$ 9.80665 metres (32.1740 ft) per s²

based upon data from *World Geodetic System 1984* (WGS-84), where g is understood to be pointing 'down' in the local frame of reference.

If it is desirable to model an object's weight on Earth as a function of latitude, one could use the following ([3] p. 41):

$$g = g_{45} - \tfrac{1}{2}(g_{\text{poles}} - g_{\text{equator}}) \cos\left(2\, lat\, \frac{\pi}{180}\right)$$

where

- g_{poles} = 9.832 metres (32.26 ft) per s²
- g_{45} = 9.806 metres (32.17 ft) per s²
- g_{equator} = 9.780 metres (32.09 ft) per s²
- *lat* = latitude, between −90 and 90 degrees

The *Earth Gravitational Model 1996* (EGM96) contains 130,676 coefficients that refine the model of the Earth's gravitational field ([3] p. 40). The most significant correction term is about two orders of magnitude more significant than the next largest term ([3] p. 40). That coefficient is referred to as the J_2 term, and accounts for the flattening of the poles, or the oblateness, of the Earth. (A shape elongated on its axis-of-symmetry, like an American football, would be called prolate.) A gravitational potential function can be written for the change in potential energy for a unit mass that is brought from infinity into proximity to the Earth. Taking partial derivatives of that function with respect to a coordinate system will then resolve the directional components of the gravitational acceleration vector, as a function of location. The component due to the Earth's rotation can then be included, if appropriate, based on a sidereal day relative to the stars (≈366.24 days/year) rather than on a solar day (≈365.24 days/year). That component is perpendicular to the axis of rotation rather than to the surface of the Earth.

`https://en.wikipedia.org/wiki/Gravitational_acceleration`

IUGG document

The observation of the Earth's gravity field by means of dedicated satellite gravity missions is a unique measurement technique for observing changes and dynamic processes that are related to mass transport the Earth's system and its components, such as the hydrosphere, cryosphere, oceans, atmosphere and solid Earth. During the last decade, with satellite gravity missions of the first generation such as GRACE and GOCE, spectacular science results and new insights could be achieved. An improved understanding of the global-state behavior of the Earth, and the quantification of dynamic processes and their coupling among the main components of the Earth system provide direct indicators of both subtle and dramatic climate change. They are an essential contribution to climate system models, and an important input for global initatives such as the Intergovernmental Panel on Climate Change (IPCC).

Beyond scientific questions, such as

- Global water cycle and the closure of the global water balance
- Sea level rise
- Global ocean circulation; mass and heat transport in the oceans
- Melting of ice sheets, e.g., Greenland, Antarctica, inland glaciers
- Dynamical processes of solid Earth
- Interaction between land and atmosphere
- Separation of natural and human-made effects on global change

a next-generation gravity mission concept will also address several practical and service applications with societal benefit (cf. Fig. E-1), such as

- Water management
- Forecasting of floods and droughts
- Climate impacts on water cycle and ice sheets
- Regional sea level changes and coastal vulnerability
- Risk assessment of natural hazards
- Globally unique definition of heights with impact on land management issues

IUGG document

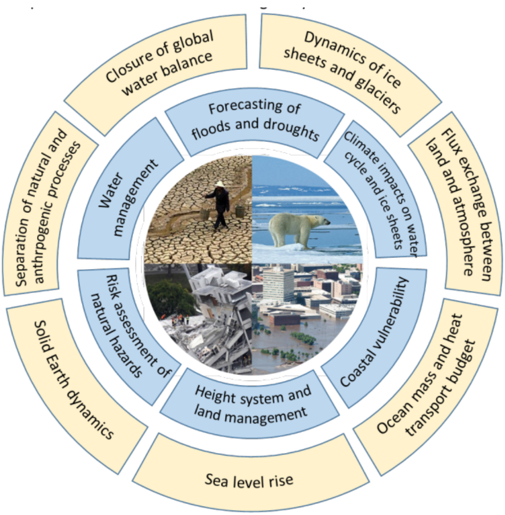
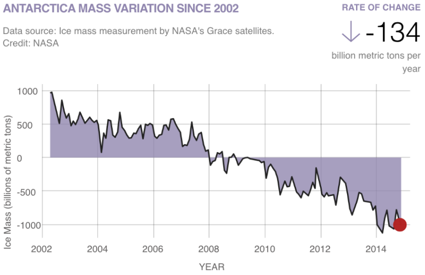
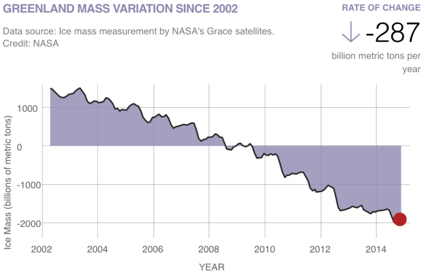
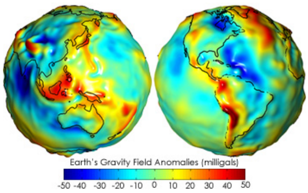


Figure E-1*: Main scientific (yellow) and societal (blue) challenges addressed by a future satellite gravity constellation.*

Using GRACE (2002-2017) satellite data







Using GRACE satellite data

Geophys. J. Int. (2007) **171,** 497–508 doi: 10.1111/j.1365-246X.2007.03556.x

FAST TRACK PAPER

Inference of mantle viscosity from GRACE and relative sea level data

Archie Paulson,* Shijie Zhong and John Wahr†

Department of Physics, University of Colorado, Boulder CO 80309, *USA. E-mail: archie.paulson@colorado.edu*

Accepted 2007 July 24. Received 2007 July 23; in original form 2006 November 30

Geophysical Journal International

Geophys. J. Int. (2018) **215,** 415–430 doi: 10.1093/gji/ggy293

Advance Access publication 2018 July 20

GJI Gravity, geodesy and tides

Exploring the uncertainty in GRACE estimates of the mass redistributions at the Earth surface: implications for the global water and sea level budgets

A. Blazquez,[1,2] B. Meyssignac,[1,2] J.M. Lemoine,[2,3,4] E. Berthier,[1] A. Ribes[5] and A. Cazenave[1]

Using GOCE (2009-2013)

Gravity Field and Steady-State **O**cean **C**irculation **E**xplorer

Mission objectives

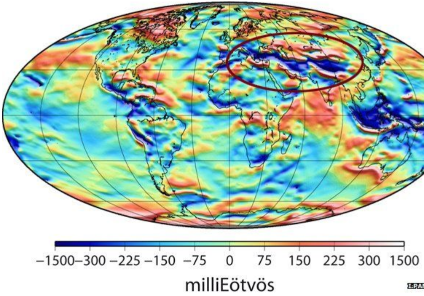
- To determine gravity-field anomalies with an accuracy of 10^{-5} m/s^2 (1 mGal). To increase resolution, the satellite flew in an unusually low orbit.
- To determine the geoid with an accuracy of 1-2 cm.
- To achieve the above at a spatial resolution better than 100 km.

Using GOCE (2009-2013)

Gravity Field and Steady-State **O**cean **C**irculation **E**xplorer

Mission objectives

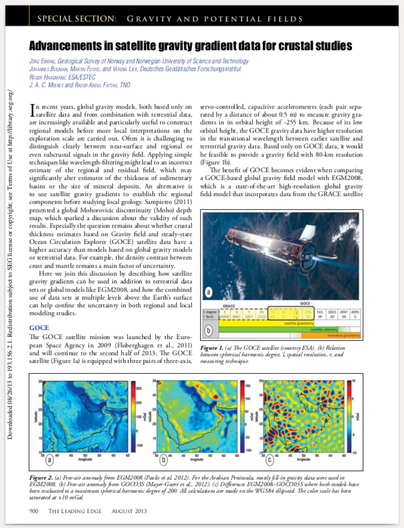
- To determine gravity-field anomalies with an accuracy of 10^{-5} m/s^2 (1 mGal). To increase resolution, the satellite flew in an unusually low orbit.
- To determine the geoid with an accuracy of 1-2 cm.
- To achieve the above at a spatial resolution better than 100 km.



Panet et al., 2014

The circled blue region reflects remnants of the old Tethys Ocean

Using GOCE data : Crustal studies (Ebbing et al., 2013)



Advancements in satellite gravity gradient data for crustal studies

Jörg Ebbing, Geological Survey of Norway and Norwegian University of Science and Technology
Johannes Bouman, Martin Fuchs, and Verena Lieb, Deutsches Geodätisches Forschungsinstitut
Roger Haagmans, ESA/ESTEC
J. A. C. Meekes and Rader Abdul Fattah, TNO

In recent years, global gravity models, both based only on satellite data and from combination with terrestrial data, are increasingly available and particularly useful to construct regional models before more local interpretations on the exploration scale are carried out. Often it is challenging to distinguish clearly between near-surface and regional or even subcrustal signals in the gravity field. Applying simple techniques like wavelength-filtering might lead to an incorrect estimate of the regional and residual field, which may significantly alter estimates of the thickness of sedimentary basins or the size of mineral deposits. An alternative is to use satellite gravity gradients to establish the regional components before studying local geology. Sampietro (2011) presented a global Mohorovicic discontinuity (Moho) depth map, which sparked a discussion about the validity of such results. Especially the question remains about whether crustal thickness estimates based on Gravity field and steady-state Ocean Circulation Explorer (GOCE) satellite data have a higher accuracy than models based on global gravity models or terrestrial data. For example, the density contrast between crust and mantle remains a main factor of uncertainty.

Here we join this discussion by describing how satellite gravity gradients can be used in addition to terrestrial data sets or global models like EGM2008, and how the combined use of data sets at multiple levels above the Earth's surface can help confine the uncertainty in both regional and local modeling studies.

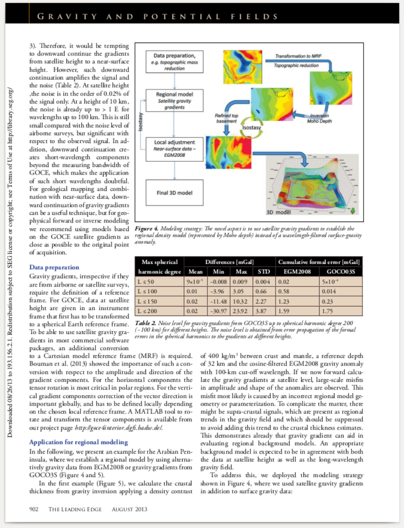
GOCE

The GOCE satellite mission was launched by the European Space Agency in 2009 (Floberghagen et al., 2011) and will continue to the second half of 2013. The GOCE satellite (Figure 1a) is equipped with three pairs of three-axis, servo-controlled, capacitive accelerometers (each pair separated by a distance of about 0.5 m) to measure gravity gradients in its orbital height of ~255 km. Because of its low orbital height, the GOCE gravity data have higher resolution in the transitional wavelength between earlier satellite and terrestrial gravity data. Based only on GOCE data, it would be feasible to provide a gravity field with 80-km resolution (Figure 1b).

The benefit of GOCE becomes evident when comparing a GOCE-based global gravity field model with EGM2008, which is a state-of-the-art high-resolution global gravity field model that incorporates data from the GRACE satellite

Figure 1. (a) The GOCE satellite (courtesy ESA). (b) Relation between spherical harmonic degree, l, spatial resolution, r, and measuring technique.

Figure 2. (a) Free-air anomaly from EGM2008 (Pavlis et al. 2012). For the Arabian Peninsula, mostly fill-in gravity data were used in EGM2008. (b) Free-air anomaly from GOCO3S (Mayer-Guerr et al., 2012). (c) Differences EGM2008–GOCO03S where both models have been evaluated to a maximum spherical harmonic degree of 200. All calculations are made on the WGS84 ellipsoid. The color scale has been saturated at ±10 mGal.



3). Therefore, it would be tempting to downward continue the gradients from satellite height to a near-surface height. However, such downward continuation amplifies the signal and the noise (Table 2). At satellite height ,the noise is in the order of 0.02% of the signal only. At a height of 10 km, the noise is already up to > 1 E for wavelengths up to 100 km. This is still small compared with the noise level of airborne surveys, but significant with respect to the observed signal. In addition, downward continuation creates short-wavelength components beyond the measuring bandwidth of GOCE, which makes the application of such short wavelengths doubtful. For geological mapping and combination with near-surface data, downward continuation of gravity gradients can be a useful technique, but for geophysical forward or inverse modeling we recommend using models based on the GOCE satellite gradients as close as possible to the original point of acquisition.

Figure 4. Modeling strategy: The novel aspect is to use satellite gravity gradients to establish the regional density model (represented by Moho depth) instead of a wavelength-filtered surface-gravity anomaly.

Max spherical harmonic degree	Differences [mGal]				Cumulative formal error [mGal]	
	Mean	Min	Max	STD	EGM2008	GOCO03S
L ≤ 50	9×10^{-5}	–0.008	0.009	0.004	0.02	5×10^{-4}
L ≤ 100	0.01	–3.96	3.05	0.66	0.58	0.014
L ≤ 150	0.02	–11.48	10.32	2.27	1.23	0.23
L ≤ 200	0.02	–30.97	23.92	3.87	1.59	1.75

Table 2. Noise level for gravity gradients from GOCO3S up to spherical harmonic degree 200 (~100 km) for different heights. The noise level is obtained from error propagation of the formal errors in the spherical harmonics to the gradients at different heights.

Data preparation

Gravity gradients, irrespective if they are from airborne or satellite surveys, require the definition of a reference frame. For GOCE, data at satellite height are given in an instrument frame that first has to be transformed to a spherical Earth reference frame. To be able to use satellite gravity gradients in most commercial software packages, an additional conversion to a Cartesian model reference frame (MRF) is required. Bouman et al. (2013) showed the importance of such a conversion with respect to the amplitude and direction of the gradient components. For the horizontal components the tensor rotation is most critical in polar regions. For the vertical gradient components correction of the vector direction is important globally, and has to be defined locally depending on the chosen local reference frame. A MATLAB tool to rotate and transform the tensor components is available from our project page *http://goce4interior.dgfi.badw.de/*.

Application for regional modeling

In the following, we present an example for the Arabian Peninsula, where we establish a regional model by using alternatively gravity data from EGM2008 or gravity gradients from GOCO3S (Figure 4 and 5).

In the first example (Figure 5), we calculate the crustal thickness from gravity inversion applying a density contrast of 400 kg/m^3 between crust and mantle, a reference depth of 32 km and the cosine-filtered EGM2008 gravity anomaly with 100-km cut-off wavelength. If we now forward calculate the gravity gradients at satellite level, large-scale misfits in amplitude and shape of the anomalies are observed. This misfit most likely is caused by an incorrect regional model geometry or parameterization. To complicate the matter, there might be supra-crustal signals, which are present as regional trends in the gravity field and which should be suppressed to avoid adding this trend to the crustal thickness estimates. This demonstrates already that gravity gradient can aid in evaluating regional background models. An appropriate background model is expected to be in agreement with both the data at satellite height as well as the long-wavelength gravity field.

To address this, we deployed the modeling strategy shown in Figure 4, where we used satellite gravity gradients in addition to surface gravity data:

Using GOCE data : lithospheric modeling (Bouman et al., 2015)

International Journal of Applied Earth Observation and Geoinformation 35 (2015) 16–30

Contents lists available at ScienceDirect

International Journal of Applied Earth Observation and Geoinformation

journal homepage: www.elsevier.com/locate/jag

ELSEVIER

GOCE gravity gradient data for lithospheric modeling

Johannes Bouman[a,*], Jörg Ebbing[b,c], Sjef Meekes[d], Rader Abdul Fattah[d], Martin Fuchs[a], Sofie Gradmann[b], Roger Haagmans[e], Verena Lieb[a], Michael Schmidt[a], Denise Dettmering[a], Wolfgang Bosch[a]

[a] *Deutsches Geodätisches Forschungsinstitut (DGFI), Alfons-Goppel-Str. 11, D-80539 Munich, Germany*
[b] *Geological Survey of Norway (NGU), Leiv Eirikssons vei 39, 7491 Trondheim, Norway*
[c] *Norwegian University of Science and Technology, 7491 Trondheim, Norway*
[d] *TNO, Princetonlaan 6, 3584 CB Utrecht, The Netherlands*
[e] *ESA/ESTEC, Keplerlaan 1, 2201 AZ Noordwijk, The Netherlands*

ARTICLE INFO

Article history:
Received 17 May 2013
Accepted 1 November 2013
Available online 12 December 2013

Keywords:
GOCE gravity gradients
Lithosphere
Moho
Heat flow determination

ABSTRACT

The Gravity field and steady-state Ocean Circulation Explorer (GOCE) is the European Space Agency's (ESA) satellite gravity mission to determine the Earth's mean gravity field. GOCE delivers gravity gradients, a new type of satellite data. We study how these data can improve modeling of the Earth's lithosphere. We discuss the use of the original GOCE gravity gradients versus the use of gravity gradients in grids at satellite altitude or close the Earth's surface and conclude that grids are easier to handle than the original data because one does not have to deal with very different error characteristics of the different gradients, given in a rotating frame at varying heights. The downward continuation to the surface enhances signal and better reflects the near-surface geology. But this does not outweigh the amplification of noise and omission errors, which is why we recommend using the field at mean satellite altitude for lithospheric modeling. The North-East Atlantic region is ideal to analyze the additional value of GOCE gravity gradients because it is a well-studied region in terms of regional geophysics. We calculated the gradient sensitivity

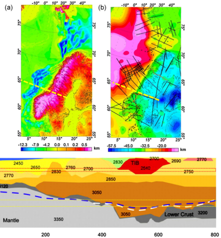


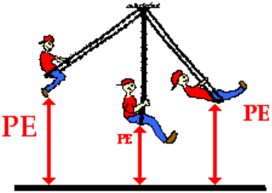
Fig. 7. Key horizons of 3D lithospheric model. (a) Top basement (after Ebbing and Olesen, 2010), (b) Moho depth, (c) crustal cross section through 3D model (Ebbing et al., 2012); depth and distance along cross section in km. Location of profile is indicated by yellow line in (a) and (b). Yellow and red box in (c) indicate depth slices for which results are shown in Fig. 8. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

Gravitational Potential Energy

- Gravitational potential energy (GPE) is the energy stored in an object as the result of its vertical position or height.
- The energy is stored as the result of the gravitational attraction of the Earth for the object.

$$PE_{grav} = mass \cdot g \cdot height$$

- To determine the gravitational potential energy of an object, a zero height position must first be arbitrarily assigned. Typically, the ground is considered to be a position of zero height.



GPE and global stress field (Ghosh et al, 2009)

Geophys. J. Int. (2009) **179**, 787–812

doi: 10.1111/j.1365-246X.2009.04326.x

Contribution of gravitational potential energy differences to the global stress field

Attreyee Ghosh,[1] William E. Holt,[2] and Lucy M. Flesch[3]

[1]*Department of Earth Science, University of Southern California,* CA 90059, *USA. E-mail: attreyeg@usc.edu*
[2]*Department of Geosciences, Stony Brook University, Stony Brook,* NY 11794-2100, *USA*
[3]*Department of Earth and Atmospheric Sciences, Purdue University, West Lafayette,* IN 47907, *USA*

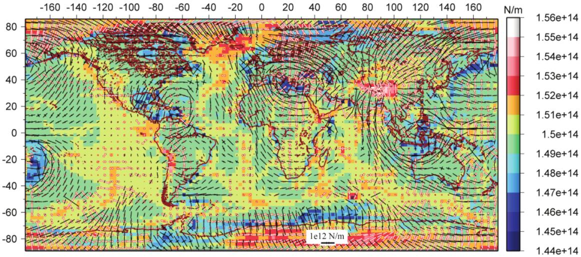


Figure 4. Global distribution of vertically integrated horizontal deviatoric stresses and GPE calculated from Crust 2.0, compensated by elevation adjustment. The range of GPE values, as well as the absolute magnitudes of deviatoric stresses, decrease compared to the uncompensated (in Fig. 2) as well as the other compensated case (Fig. 3), but the overall pattern remains similar to that in Figs 2 and 3. Because compensation is achieved via elevation adjustment, Fig. 4 is the theoretical response of lithosphere from internal buoyancies, with the influence of dynamic topography removed. Therefore, the GPE differences for young versus old oceanic lithosphere in this model arise only from the elevation and density changes associated with lithospheric cooling.